Research Paper

Codify a Three echelon inventory control model in terms of inflation with allowed shortage for a deterioration items

Mohammad Hassan Damyad *, Davoud Jafari *

*Department of Industrial engineering, Parand Branch, Islamic Azad University, Parand, Iran

ABSTRACT

In this research initially, trying to find a model for inventory control system in a three-echelon supply chain for one-product system composed of levels of production, warehouse and seller and then will be trying to find optimal point in mentioned model. Model condition has been under inflation and has been considered for a commodity in which shortage is allowed and production has been deteriorating. With assumption be stable lead time and rate of demand and production and deterioration factor, obtain the overall cost function and restrictions for mentioned state. The impact of inflation with exponential function is on the price of units after that because of complex non-linear equation obtained, possibility of solving is impossible through classical methods. So we use MATLAB software for the numerical solution, sometime in numerical cases MATLAB can’t solve in routine time or can’t solve so we proposed the simulated annealing metaheuristic algorithm for these cases.

© 2021 IJIE. All rights reserved.

1. Introduction

In 1915 the simplest Economic Ordering Quantity (EOQ) was introduced by someone named “Harris F.W” at the Institute of Restynghaves. (Harris, 1915). Misra has examined EOQ model for Non deterioration items with fixed and variable rates. (Misra, 1975). Aggarwal and Bahari provided a model in which fixed rate of corruption and rate of demand in which decrease had been assumed with a negative exponential function and also had assumed production rate fixed but if in different periods has been changeable (Aggarwal and Bahari, 1991). in 1964 Hadley consider the
time value of money, He calculated order size with average annual cost method and discounted costs and proved that almost cost obtained from both methods are equal (Hadley, 1964). Buzacott in 1975 was the first person who raised Inventory control issues with regard to inflation and he concluded that the EOQ model should include changes by taking into account inflation (Buzacott, 1975). Fattahi and colleagues in 2013 have achieved algorithm for inventory control model with probabilistic demand and influenced by amount of shortages and have considered demand function affected by the shortages and have considered a normal probability demand function and have solved model with quick freezing and genetic algorithms (Fattahi et al., 2013). In 2015 a development method was done to calculate the optimal mode in multi-parametric mode for economic order quantity model (Salvatore et al., 2015). Jindal have discussed on a specific inventory model with discounted price shortage and controllable Lead Time. (Jindal and Solanki, 2014) after that amount of inventory cost affected by the inflation has been assumed in the form of a single seller and single buyer Supply Chain in the form of a EOQ model and has been resolved in both cases which lead time has normal distribution and free distribution (Jindal, 2016). Damyad provide a three echelon model seller, warehouse and manufacturing under the inflation and investigate for corruptible and non-deterioration items in cases of allowed and non-allowed shortage (Damyad, 2016)

2. Research model

Discussed model in this research includes three levels of manufacturer, warehouse and seller such that commodity production is sent to warehouse and in required time is sent for the seller the relationship between these levels has been shown in Figure 1.

![Figure 1: Relationship between these levels](image)

**Notation:** In Table 1 introduces used symbols in this research which will be used of it in describing the model and relevant equations
<table>
<thead>
<tr>
<th>Definition</th>
<th>Symbol</th>
<th>Row</th>
<th>Definition</th>
<th>Symbol</th>
<th>Row</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit cost of product production at the beginning of first cycle</td>
<td>$u$</td>
<td>19</td>
<td>Total Holding Cost</td>
<td>$THC$</td>
<td>1</td>
</tr>
<tr>
<td>The cost of Order Product in first cycle seller level</td>
<td>$C_v$</td>
<td>20</td>
<td>Total Ordering Cost</td>
<td>$TOC$</td>
<td>2</td>
</tr>
<tr>
<td>The unit cost of product orders in the first cycle warehouse</td>
<td>$C_s$</td>
<td>21</td>
<td>Total Shortage Cost</td>
<td>$TSC$</td>
<td>3</td>
</tr>
<tr>
<td>Rate of deterioration</td>
<td>$\theta$</td>
<td>22</td>
<td>Total Deterioration Cost</td>
<td>$TDC$</td>
<td>4</td>
</tr>
<tr>
<td>inflation rate</td>
<td>$\delta$</td>
<td>23</td>
<td>The cost of holding at the beginning of the first cycle of seller level</td>
<td>$h_v$</td>
<td>5</td>
</tr>
<tr>
<td>Interest rate</td>
<td>$\delta$</td>
<td>24</td>
<td>The cost of product manufacturing in the $i$ cycle</td>
<td>$MC_i$</td>
<td>6</td>
</tr>
<tr>
<td>lead time Product (constant number)</td>
<td>$L$</td>
<td>25</td>
<td>The cost of product holding in the $i$ cycle</td>
<td>$HC_i$</td>
<td>7</td>
</tr>
<tr>
<td>The unit cost of launching in production level in the first cycle</td>
<td>$C_{se}$</td>
<td>26</td>
<td>Amount of inventory in the $i$ cycle</td>
<td>$H$</td>
<td>8</td>
</tr>
<tr>
<td>Interest rate of net of inflation ($\delta = \hat{\delta} - \delta$)</td>
<td>$\delta$</td>
<td>27</td>
<td>Reorder point (decision variables)</td>
<td>$r$</td>
<td>9</td>
</tr>
<tr>
<td>lead time Product (constant number)</td>
<td>$L$</td>
<td>28</td>
<td>amount of each Order (decision variables)</td>
<td>$Q$</td>
<td>10</td>
</tr>
<tr>
<td>The total cycles at the level of seller within a year</td>
<td>$N$</td>
<td>29</td>
<td>The number of seller cycles are within a production cycle (decision variables)</td>
<td>$m$</td>
<td>11</td>
</tr>
<tr>
<td>The total number of cycles at the level of warehouse for one year</td>
<td>$n$</td>
<td>30</td>
<td>Demand Product rate (constant number)</td>
<td>$D$</td>
<td>12</td>
</tr>
<tr>
<td>Time of a cycle at the level of seller</td>
<td>$T$</td>
<td>31</td>
<td>Production rate (constant number and $P &gt; D$)</td>
<td>$P$</td>
<td>13</td>
</tr>
<tr>
<td>Time</td>
<td>$t$</td>
<td>32</td>
<td>Backlog coefficient ($0 &lt; \beta &lt; 1$)</td>
<td>$\beta$</td>
<td>14</td>
</tr>
<tr>
<td>The time interval from reaching order in each cycle seller until re-order</td>
<td>$t^*$</td>
<td>33</td>
<td>The unit cost of dealing with the shortages at the beginning of the first cycle</td>
<td>$\pi$</td>
<td>15</td>
</tr>
<tr>
<td>When in production cycle (its time is $m$ times of a seller’s cycles)</td>
<td>$\tilde{t}$</td>
<td>34</td>
<td>The unit cost of shortages kind of lost sales at the beginning of the first cycle</td>
<td>$\pi_0$</td>
<td>16</td>
</tr>
<tr>
<td>Definition</td>
<td>Symbol</td>
<td>Row</td>
<td>Definition</td>
<td>Symbol</td>
<td>Row</td>
</tr>
<tr>
<td>---------------------------------------------------------------------------</td>
<td>--------</td>
<td>-----</td>
<td>---------------------------------------------------------------------------</td>
<td>--------</td>
<td>-----</td>
</tr>
<tr>
<td>Unit cost of deterioration at the beginning of first cycle in product echelon</td>
<td>( \omega_p )</td>
<td>35</td>
<td>Unit cost of deterioration at the beginning of first cycle in vendor echelon</td>
<td>( \omega_v )</td>
<td>17</td>
</tr>
<tr>
<td>Unit cost of deterioration at the beginning of first cycle in storage echelon</td>
<td>( \omega_s )</td>
<td>18</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Assumptions:** Also research Assumptions is described in Table 2.

Table 2: Assumptions

<table>
<thead>
<tr>
<th>Assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Demand is always a constant number ( D ) in all cycles.</td>
</tr>
<tr>
<td>2. Production rate is always constant number ( P ) and always is more than rate of demand. ( (P &gt; D) )</td>
</tr>
<tr>
<td>3. Model is for a Single-product process.</td>
</tr>
<tr>
<td>4. Commodity is deteriorating with constant deteriorate factor ( \theta ) .</td>
</tr>
<tr>
<td>5. Allowed shortages is the kind of Back Order and Back Logging that if is amount of deducted ( \beta ) of the total shortages As Backlog. It is obvious that lost sales will be to amount ( 1 - \beta ) ( (0 &lt; \beta &lt; 1) )</td>
</tr>
<tr>
<td>6. Lead Time is constant.</td>
</tr>
<tr>
<td>7. During time horizon is one year.</td>
</tr>
<tr>
<td>8. The cost of Faced with shortages maintenance, purchasing and order each of them are as a function of the rate of inflation in each period.</td>
</tr>
<tr>
<td>9. Time value of money and interest rate and inflation has been considered With fixed rate.</td>
</tr>
<tr>
<td>10. Model has been composed trihedral of manufacturer, warehouse and seller.</td>
</tr>
</tbody>
</table>

2.1. Seller Echelon

At this Level of amount of inventory is declining influences the demand rate and deterioration rate until amount of balance reaches to order point and at the same time, order command will be issued to amount of \( Q \) \( (R \) is reorder point). But these order is collected in \( L \) units of time after it. At this moment, amount of inventory under the influence of demand reduced with a fixed rate \( D \) of the total inventory demand rate and deterioration rate. Until the moment \( (t_A) \) amount of inventory reaches zero. After these
moment is not able to meet the demand so there is demand to rate of D without response until at moment (L) order enters the system to amount of determined. After that amount of inventory that is compensate for the shortage deducted from it’s and rest enter the system as initial inventory of next cycle. Depending on Backlog coefficient is constant according to assumption. So β coefficient amount of the total shortage amount is as backlog and this amount deducted from amount of received order. So coefficient (1-β) of the total amount of shortages is as lost sales. This procedure is also happening in the next cycles. To better explain of Figure2 the process is as follows.

![Figure 2: Seller Echelon](image)

To determine inventory Quantity we separate every cycle into two part. First part is from 0 to \( t^* + t_A \) of time that we don’t have shortage so variable \( I_1 \) defines inventory levels and second part is from \( t^* + t_A \) to T of time so variable \( I_2 \) defines inventory in this part.

Inventory levels and shortage in each cycle in a period of time between the beginning of cycle until the completion of inventory and from completion of inventory to receipt of amount of order is as follows.

\[
\frac{dI_1(t)}{dt} = -D - \theta I_1(t)
\]

\[
\frac{dI_2(t)}{dt} = -D
\]

So we will had:
\[ I_1(t) = \left[ \frac{D}{\theta} + I(0) \right] e^{-\theta t} - \frac{D}{\theta} \]

\[ I_2(t) = I(0) - D \]

We can calculate quantity of \( t_A \)

\[ t_A = \frac{1}{\theta} \ln \left( \frac{D + \theta r}{D} \right) \]

The amount of commodity beginning of each cycle is obtained through the following:

\[ Q^* = Q - \beta S = Q - \beta D \left[ L - t_A \right] \]

The period of time in every cycle is constant and specified

\[ T = L + t^* \]

So we can calculate quantity of \( t^* \)

\[ t^* = \frac{1}{\theta} \ln \left( \frac{\theta Q^* + D}{\theta r + D} \right) \]

**A: Shortage Costs**

To obtain the amount of shortage cost should be achieved shortage amount in each cycle.

\[ TSC = SC_1 + SC_2 + \cdots + SC_n \]

Shortage amount is started in each cycle in time, after reaching amount of inventory to zero and continues until the end of the cycle. also know that shortage amount increasing based on \( I_2(t) \) function until the end of cycle and shortage amount in fact is equal to its mathematical expectation that because here does not have amount of shortage of probabilistic model, exact amount of it is to be calculated by integral.

\[ S_i = E(x - r)^+ = \int_0^{L-t_A} I_1(t) \, dt \]
According to what was said, unit cost faced with a shortage is considered for all shortage amounts but amount of cost of lost sales is only imposed by amount of sales lost to System. It is obvious that shortage amount in the state of backlog that is compensated in the next cycle does not impose such cost to system with what was stated, can the total cost of dealing with the shortage system in a cycle calculate as follows:

\[
TSC_v = D[\pi + \pi_0(1 - \beta)]\left[ L - \left( \frac{1}{\theta} \ln \left( \frac{D + \theta r}{D} \right) \right) \right] \left( 1 + e^{\delta(T)} + e^{2\tau\delta} + \cdots + e^{(N-1)\tau\delta} \right) \\
= D[\pi + \pi_0(1 - \beta)]\left[ L - \left( \frac{1}{\theta} \ln \left( \frac{D + \theta r}{D} \right) \right) \right] \left( \frac{1 - e^{NT\delta}}{1 - e^{\tau\delta}} \right)
\]

**B. Holding Costs**

To obtain holding cost should have amount of inventory along with holding time in each cycle. Initially earns amount of inventory in the first cycle, also know that in all cycles amount of inventory is identical.

\[
H = \int_{t^*}^{t^*+t_A} I_1(t) \, dt \\
H = \left[ Q - \beta D[L - t_A] \right] - \frac{D}{\theta^2} \ln \left( \frac{D + \theta Q - \beta D\theta[L - t_A]}{D} \right)
\]

Maintenance costs, taking into account amount of inflation of each cycle will be as follows.

\[
THC_v = H h_v \left( \frac{1 - e^{NT\delta}}{1 - e^{\tau\delta}} \right)
\]

**C. deterioration Costs**

To obtain deterioration cost should have quantity of deterioration inventory along time in each cycle. We assumed deterioration rate is constant \( \theta \) . The quantity of deterioration obtain from initially inventory minus demand that supply in same time.

\[
Z = Q^* - D(t^* + t_A)
\]
\[ TDC_v = Z \omega_v \left( 1 + e^{\delta(T)} + e^{2\delta T} + \cdots + e^{(N-1)\delta T} \right) = Z \omega_v \left( \frac{1 - e^{NT\delta}}{1 - e^{\delta T}} \right) \]

**D. Ordering Costs**

Ordering costs is considered at the beginning of the cycle that changes according to inflation in each cycle we assume ordering costs of the first cycle \( C_v = C_{v1} \) is obvious that ordering costs in the next cycle is updated with function of inflation rate and the value of money.

So amount of total ordering cost is as follows:

\[ TOC = OC_{v1} + OC_{v2} + \cdots + OC_{vn} \]

\[ = C_v \left( 1 + e^{\delta(T)} + e^{2\delta T} + \cdots + e^{(N-1)\delta T} \right) \]

\[ = C_v \left( \frac{1 - e^{NT\delta}}{1 - e^{\delta T}} \right) \]

According to the above was mentioned except inventory costs could be gained the total cost of inventory in the state inflation at the level of Seller.

\[ TIC_v(Q, r, N) = TSC_v + THC_v + TDC_v + TOC_v \]

\[ = \left[ \left( D[\pi + \pi_0(1 - \beta)] \right) \left[ L - \left( \frac{1}{\theta} \ln \left( \frac{D + \theta r}{D} \right) \right) \right] + Z \omega_v + H h_v + C_v \right] \left( \frac{1 - e^{NT\delta}}{1 - e^{\delta T}} \right) \]

**2.2. Warehouse Echelon**

Manufactured goods are kept in warehouse and at specified times will be sent to level of seller. This process is as cycle such a way that at the beginning of each cycle of warehouse amount of \( M \) product enter warehouse of the production level and after that during each cycle seller of amount of \( Q \) product is sent from the warehouse to level of seller and this procedure continues until the inventory reaches zero. Duration of cycle products deteriorate with constant factor. We see that the duration of each cycle of warehouse level is \( m \) times the cycle of seller level .In Figure 3- amount of inventory has been shown at this level.

108
Figure 3 inventory in interfaces Warehouse level

At this level, inventory is not under impact of any rate to change, Just in specified time periods amount of it will be sent to the seller and it deteriorated with constant factor Therefore, amount of inventory changes in period is as follows.

\[
\frac{dl_3}{dt} = -\theta
\]

Quantity of M reached with identify inventory function.

\[
M = Q \left( 1 - e^{mT\theta} \right) \left( 1 - e^{-\theta T} \right)
\]

So we can calculate costs of this echelon.

**A. Holding Costs**

To determine the holding costs should be identified amount of inventory in holding time. To achieve this goal in Warehouse cycle, amount of available commodity is \( M \) at the beginning of each cycle. After every \( T \) units of time is reduced to amount of \( Q \) of Commodity. So amount of commodity in time obeys according to the following equation.

Therefore, the inventory - time commodity in a warehouse cycle is as follows

\[
\sum_{i=1}^{m} l_3^i = M \left( \frac{1 - e^{-m\theta T}}{1 - e^{-\theta T}} \right) - \sum_{i=1}^{m-1} (m - i) Q e^{-i\theta T}
\]
In the case of existence of inflation, the unit cost of holding of the cycle changes to another cycle. So after adjusting for inflation, the total holding cost at the warehouse level will be as follows

\[ THC_s = \frac{1 - e^{-\theta T}}{\theta} h_s \left( \sum_{i=1}^{m} I_i^1(0) - mQ \right) \left( 1 + e^{mT\delta} + e^{2mT\delta} + \cdots + e^{(n-1)mT\delta} \right) \]

\[ = \frac{1 - e^{-\theta T}}{\theta} h_s \left( \sum_{i=1}^{m} I_i^1(0) - mQ \right) \left( \frac{1 - e^{nmT\delta}}{1 - e^{mT\delta}} \right) \]

**B. Deterioration Costs**

Quantity of products that deteriorate at every cycle reaches from difference amount of inventory at the beginning of cycles and inventory that exit from storage. So function of deterioration is:

\[ M - mQ \]

Costs of deterioration in the Cycle is:

\[ \omega_s (M - mQ) \]

*If we have costs of first cycle, we will calculate costs of every cycle with inflation by below function*

\[ TDC_s = (M - mQ) \omega_s \left( \frac{1 - e^{nmT\delta}}{1 - e^{mT\delta}} \right) \]

**C. Ordering Costs**

If for any time of sending commodity from manufacturing to warehouse costs are paid at the rate unit \( C_s \). Then amount of total ordering cost will be as follows

\[ TOC_s = C_s \left( 1 + e^{mT\delta} + e^{2mT\delta} + \cdots + e^{(n-1)mT\delta} \right) = C_s \left( \frac{1 - e^{nmT\delta}}{1 - e^{mT\delta}} \right) \]

According to the above calculations in amount of total cost of inventory at interface level warehouse is as follows
\[ TIC_s(Q,T,m,n) = THC + TDC + TOC = \left( \frac{1 - e^{-\theta T}}{\theta} \right) h_s \left( \sum_{i=1}^{m} I_i^s(0) - mQ \right) + Q \omega_s \left( \frac{1 - e^{mT\theta}}{1 - e^{\tau\theta}} \right) - m \right) + C_s \left( \frac{1 - e^{nmT\delta}}{1 - e^{mT\delta}} \right) \]

2.3. Manufacturing level

In level of manufacturing of commodity at a time when the car started to produce will be added to amount of inventory and in time to stop all commodity produced will be sent to the warehouse. So that commodity does not remain at the manufacturing level. If production be at fixed rate \( P > D \) and a cycle of this level is considered \( M \). at the moment of \( t_s \) machine Start and started to produce and the machine is turned off at the moment of \( mT \) and all inventory is transferred to the warehouse. This process has been shown in Figure -4.

![Diagram of inventory time in the level of manufacturing](image-url)

Figure 4. Diagram of inventory time in the level of manufacturing
According to the above figure, Can be knew existing equation in the manufacturing cycle influenced by production rate. So according to what to state about inventory flow will have:

\[
\frac{dI_4}{dt} = -\theta I_4(t) + P
\]

\[
I_4(t) = I(0)e^{-\theta t} + \frac{P}{\theta}(1 - e^{-\theta t})
\]

By having inventory relations in this level know that at the moment of \(t_s\) amount of inventory is zero. So with this issue and at the \(mT\) moment, amount of inventory reaches \(M\) can calculate the amount of \(t_s\) time.

\[
mT - t_s = \bar{t} = \frac{1}{\theta} \ln \left( \frac{P (1-e^{\theta \bar{t}})}{P (1-e^{\theta t}) - Q \theta (1-e^{mT \theta})} \right)
\]

**Production echelon costs**

**A. Set-up Costs**

During Set-up the machine amount of costs should be spent and because we are in inflation space. This is made price change in each cycle so will have:

\[
TSeC = C_{se} \left( 1 + e^{mT \delta} + e^{2mT \delta} + \cdots + e^{(n-1)mT \delta} \right) = C_{se} \left( \frac{1 - e^{nmT \delta}}{1 - e^{mT \delta}} \right)
\]

**B. Production costs**

To produce any Commodity, Production costs have specific amount. We know that in mechanical production process activities for \(\bar{t}\). Therefore, the total production is obtained as follows with the symbol \(TP\).

\[
TP = P(\bar{t})
\]

In the case that there will be inflation, the cost of production in all the cycles is as follows
\[ TP_eC = m \ Q \ u(1 + e^{mT\delta} + e^{2mT\delta} + \cdots + e^{(n-1)mT\delta}) = m \ Q \ u\left(\frac{1 - e^{nmT\delta}}{1 - e^{mT\delta}}\right) \]

**C. Holding Costs**

As was mentioned, manufactured commodities will be sent to the warehouse at the end of production cycle. But in a period of production, amount of commodity is also kept at this level that should also determine this amount and their costs. Amount of inventory-time a production cycle will be calculated by the following equation.

\[ H = \int_{0}^{\bar{t}} I_4(t) \, dt = \frac{P}{\theta}\left[\bar{t} + \frac{1}{\theta}(e^{-\bar{t}\theta} - 1)\right] \]

By taking into account the impact of inflation and the amount of time-inventory are equal together in all cycles, The holding costs in production cycle is as follows.

\[ THC_p = \frac{P}{\theta}\left[\bar{t} + \frac{1}{\theta}(e^{-\bar{t}\theta} - 1)\right]\left(\frac{1 - e^{nmT\delta}}{1 - e^{mT\delta}}\right)h_p \]

**D. Deterioration Costs**

Quantity of deterioration can calculate with below equation

\[ DQ = P(\bar{t}) - M \]

If we have inflation in system, we will calculate costs of deterioration by:

\[ TDQ_p = \left[ P\bar{t} - M \right] \omega_p \left(1 + e^{mT\delta} + e^{2mT\delta} + \cdots + e^{(n-1)mT\delta}\right) \]

According to the above calculations, total inventory cost in production levels by adjusting for inflation will be as follows.
\[ TIC_p(m,n,T,Q) = TS_eC_p + TPeC_p + THC_p \]
\[ (C_{se} + m Q \ u + \frac{m^2 Q^2}{2p} \ h_p) \left( 1 - e^{nmT\delta} \right) \]

So total inventory cost in the trihedral system is as follows:

\[ TIC(Q, r, N, m, n) = TIC_v(Q, r, N) + TIC_s(Q, r, n, m) + TIC_p(Q, r, n, m) \]
= \[ [( D[\pi + \pi_0(1 - \beta)]) \ ( L - t_A ) + H_1 h_v + Z_\omega_v + C_v] \left( 1 - e^{NT\delta} \right) \]
+ \[ \left[ \frac{(1 - e^{-\theta T})}{\theta} \right] \left( \sum_{i=1}^{m} l_i^4(0) - mQ \right) + Q_\omega \left( \left( 1 - e^{mT\theta} \right) - m \right) + C_s + C_{se} - \frac{P}{\theta^2} h_f \]
+ \[ P \left[ u + \frac{h_f}{\theta} + \omega_f \right] \bar{t} + \frac{P}{\theta^2} h_f e^{-\bar{t}T} - M_\omega \left( 1 - e^{mT\delta} \right) \]

2.4 Constraints

According to figures provided at different levels of model and also conditions referred to in them in each level, has a series of limitations that are expressed in below in breakdown of each level.

A. Constrains period of one year

As is evident, the number of cycles of level seller is equal to \( N \) and the duration of each cycle is \( T \) and cycles of warehouse and manufacturing level \( n \) cycle is implemented in a year and each cycle time is \( mT \) So will have:

\[ NT = mnT = 1 \rightarrow \frac{T}{N} = \frac{1}{mn} \]

B. Constraint related to the period from order point to out of inventory point (\( t_A \))
The assumption is on shortage in per cycle. So to achieve this, it is necessary, completion time of commodity is maximum equal to time of reaching order.

\[ t_A \leq L \]
\[ r < \frac{1}{\theta} (De^{L\theta} - D) \]

**C. Constraints related to the period from entering order up to reaching to order point (t*)**

As previously was mentioned, time of a cycle seller follows of \( T = t^* + L \) function. Therefore, can conclude from the above equation

\[ Q_{max} = e^{\theta(1-L)(\theta r + D) - D} + \beta D \left[ L - \frac{1}{\theta} \ln \left( \frac{D + \theta r}{D} \right) \right] \]

**D. Minimum order quantity Constrain**

Minimum order quantity, for that be maintained relations governing on model, the minimum Q amount is as follows:

\[ Q_{min} = r + \beta D \left[ L - \frac{1}{\theta} \ln \left( \frac{D + \theta r}{D} \right) \right] \]

**E. Constrain the number of seller cycle in warehouse cycle**

According to the equality time of planning at different levels that is for one year according to following relation, amount of maximum warehouse and manufacturing cycle time creates following limitations for us.

\[ nmT = NT \quad \rightarrow \quad m = \frac{1}{nT} \quad \quad 1 \leq m \leq \frac{1}{T} \]

**F: limitation related to production time in each cycle**
Production cycle includes a time period machine uptime and a machine rest time. Maximum machine uptime if can be achieved that machine rest time be zero so this limits is as follows:

\[ \bar{t} < mT \]
\[ \frac{Q\theta}{(e^{\theta\bar{t}} - 1)} e^{mT\theta} < p \]

It should be noted that the maximum period cycle is \( T = 1 \) on the other hand we initially assumed production rate greater than the rate of consumption so mentioned limits is surplus.

**G: Conclusion of model general limitation**

From explanation was previously mentioned, Limitations of model is altogether as following form.

\[
0 \leq r < \frac{1}{\theta}(De^{L\theta} - D)
\]
\[
r + \beta D \left[ L - \frac{1}{\theta} \ln \left( \frac{D + \theta r}{D} \right) \right] < Q < \frac{e^{\theta(1-L)}(\theta r + D) - D}{\theta} + \beta D \left[ L - \frac{1}{\theta} \ln \left( \frac{D + \theta r}{D} \right) \right]
\]
\[
1 \leq m \leq \frac{1}{T} \quad \rightarrow \quad 1 \leq m \leq \frac{1}{L + \frac{1}{\theta} \ln \left( \frac{\theta Q - \theta \beta D L + \beta D \ln(\frac{\theta r + D}{D}) + D}{\theta r + D} \right)}
\]

**3. Solving method of model**

It can be seen that the solution of non-linear function obtained from TJC from classic method is very complex and practically impossible and differentiation from mentioned function is hard based on decision varies. That is why to find answers from MATLAB software to full count the different scenarios we will use three decision variables.

**Parameter values**
In the studied model, consider Model parameter values in accordance with the following table

<table>
<thead>
<tr>
<th>D</th>
<th>P</th>
<th>β</th>
<th>π</th>
<th>π₀</th>
<th>hᵣ</th>
<th>hₛ</th>
<th>h₟</th>
<th>u</th>
<th>Cᵥ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>5000</td>
<td>0.2</td>
<td>10</td>
<td>100</td>
<td>20</td>
<td>30</td>
<td>25</td>
<td>500</td>
<td>1200</td>
</tr>
<tr>
<td>Cₛ</td>
<td>ωᵣ</td>
<td>ωₛ</td>
<td>ω₟</td>
<td>Cₛₑ</td>
<td>L</td>
<td>θ</td>
<td>δ</td>
<td>δ</td>
<td>δ</td>
</tr>
<tr>
<td>1000</td>
<td>1400</td>
<td>1200</td>
<td>1000</td>
<td>3500</td>
<td>5/365</td>
<td>0.1</td>
<td>0.15</td>
<td>0.2</td>
<td>-0.05</td>
</tr>
</tbody>
</table>

By using mentioned software, optimal value is as follows

<table>
<thead>
<tr>
<th>Q</th>
<th>r</th>
<th>m</th>
<th>TIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0</td>
<td>30</td>
<td>295614</td>
</tr>
</tbody>
</table>

Some time may be in specified condition we can’t use explained method because of MATLAB solution take long time so we propose a metaheuristic method

**Simulated annealing Method**

A. *In this case study quantity of diction variable r obtain from constrain as*

\[ 0 \leq r \leq 13 \]

*So we calculate with everyone from zero to 13*

B. **MARCOV CHAIN**

Quantity of diction variable Q is between \( Q_{\text{min}} \) and \( Q_{\text{max}} \) so we define this Q as temperature variable in Simulated annealing and it primary Quantity is \( Q_{\text{max}} \) reduced by 0.005 (constant factor)

C. **METRO POLIS rule**

In every iteration we have \( TIC(Q, r, m) \). It will be better if it reduce so we store Quantity of them and if it doesn’t reduce, we will accept them with probability of generated stochastic number between 0 and 1 smaller than \( \frac{\Delta TIC}{r} \).

Optimum point that reached from this method is
Error of this method is about 10% so we use exactly method in MATLAB unless it cannot solve in the best time

Table 3: Sensitivity Analysis of Parameters

<table>
<thead>
<tr>
<th>TIC</th>
<th>r</th>
<th>Q</th>
<th>m</th>
<th>W_f</th>
<th>W_s</th>
<th>W_v</th>
<th>h_f</th>
<th>h_s</th>
<th>h_v</th>
<th>π_0</th>
<th>π</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>295614</td>
<td>0</td>
<td>3</td>
<td>30</td>
<td>100</td>
<td>120</td>
<td>1400</td>
<td>25</td>
<td>30</td>
<td>20</td>
<td>100</td>
<td>10</td>
<td>0.2</td>
</tr>
<tr>
<td>551606</td>
<td>13</td>
<td>239</td>
<td>1</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>3000</td>
<td>1000</td>
<td>&quot;</td>
</tr>
<tr>
<td>606461</td>
<td>13</td>
<td>128</td>
<td>1</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>1000</td>
<td>900</td>
<td>500</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

4. Conclusion

Raised about investigated numerically can be seen that, point of order whatever be less is more appropriate and also amount of order is also small amount it is obvious that optimal value decision three variables \((Q, r, m)\) depend on other fixed parameters and fixed costs which the following table show the influence of some of them in decision variables and objective function.

By examining the above table can be seen that, Optimality is sensitive than the unit cost of dealing with shortages and Whatever this amount is higher than holding costs, model has desire to save more and more order quantity and if this ratio is reverse, model has desire to save less and amount of order is reduced.

The obtained model in terms of inflation for a three echelon offer to us amount of optimal ordering and optimal point of order, however, can expand this model by changing in the model adapted to different conditions and added domain of using it.

Reference


Fattahi, Parviz and Turkmen, A. Kheirkhah, A Fatah Ullah, M. 2013. Introduce an algorithm to Model inventory control (r, Q) with function of potential demand and influenced by shortage amount". Production and operations management - fourth period - Number 1 - Spring and Summer 2013- PP21-38.


